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TRIGONOMETRÍA

DIRIGIDA

SEMANA 18

PROBLEMA 1

Para transmitir una señal $f(x)=\text{sen}2x$ se superpone positivamente con $g(x)=\text{sen}6x$ resultando una tercera señal $h(x)=2\text{sen}4x$ determine la suma de valores de x en el intervalo de $0 \le x \le \frac{\pi}{2}$.

A) $\frac{3\pi}{4}$ B) $\frac{2\pi}{3}$ C) π

D) $\frac{4\pi}{3}$ E) 2π

$$\text{Sen}A + \text{Sen}B = 2\text{sen}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$ab = ac \longrightarrow a = 0 \quad \vee \quad b = c$$

Resolución

Dato: $g(x) + f(x) = h(x)$

$$\text{sen}6x + \text{sen}2x = 2\text{sen}4x \; ; \; 0 \le x \le \frac{\pi}{2}$$

$$2\,\text{sen}4x\cos 2x = 2\,\text{sen}4x$$

- $\text{Sen}4x = 0$

$$4x = n\pi \; ; n \in \mathbb{Z} \longrightarrow x = \frac{n\pi}{4} \; ; \; n \in \mathbb{Z}$$

$$\longrightarrow x = 0; \frac{\pi}{4}; \frac{\pi}{2}$$

- $\cos 2x = 1$

$$2x = 2n\pi \; ; \; n \in \mathbb{Z} \longrightarrow x = n\pi \; ; \; n \in \mathbb{Z}$$

$$\longrightarrow x = 0$$

$$\longrightarrow CS = \left\{0; \frac{\pi}{4}; \frac{\pi}{2}\right\}$$

$$\therefore \text{Suma de Soluciones} = 0 + \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

PROBLEMA 2

Halle la suma de soluciones de la ecuación

$$\frac{\operatorname{sen}3x}{\cos x} - 2\operatorname{sen}x - 2\operatorname{sen}2x = 0$$

Donde $0 \le x \le 2\pi$.

A) 3π B) $\frac{\pi}{2}$ C) π

D) 2π E) 5π

$\operatorname{Sen}2\alpha = 2\operatorname{Sen}\alpha\cos\alpha$

$2\operatorname{Sen}A\cos B = \operatorname{Sen}(A+B) + \operatorname{Sen}(A-B)$

$a \cdot b = 0 \longrightarrow a = 0 \ \vee \ b = 0$

Resolución

- $\cos x \neq 0 \longrightarrow x \neq \frac{\pi}{2}; \frac{3\pi}{2}$

- $\operatorname{Sen}3x - 2\operatorname{Sen}x\cos x - 2\operatorname{Sen}2x\cos x = 0; \ 0 \le x \le 2\pi$

$\operatorname{Sen}3x - \operatorname{Sen}2x - (\operatorname{Sen}3x + \operatorname{Sen}x) = 0$

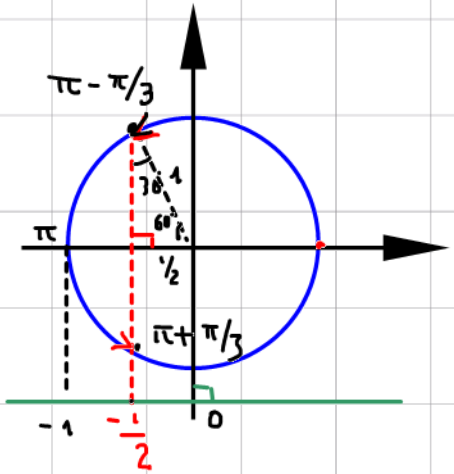
$-\operatorname{Sen}2x - \operatorname{Sen}x = 0 \longrightarrow \operatorname{Sen}2x + \operatorname{Sen}x = 0$

$2\operatorname{Sen}x\cos x + \operatorname{Sen}x = 0 \longrightarrow \operatorname{Sen}x(2\cos x + 1) = 0$

i) $\operatorname{Sen}x = 0 \longrightarrow x = 0; \pi; 2\pi$

ii) $\cos x = -\frac{1}{2} \longrightarrow x = \frac{2\pi}{3}; \frac{4\pi}{3}$

$\longrightarrow CS = \left\{0; \frac{2\pi}{3}; \pi; \frac{4\pi}{3}; 2\pi\right\}$



$\therefore$ Suma de soluciones $= 5\pi$

PROBLEMA 3

Determine la solución general de la ecuación
$\tan^3x-\tan^2x-3\tan x+3=0;\ \tan x>1$

A) $\left\{n\pi\pm\frac{\pi}{3};n\in\mathbb{Z}\right\}$

B) $\{4n\pi;n\in\mathbb{Z}\}$

C) $\left\{2n\pi\pm\frac{\pi}{3};n\in\mathbb{Z}\right\}$ → $x=\pm\frac{\pi}{3}$ → $\tan x=\pm\sqrt{3}$

D) $\left\{n\pi+\frac{\pi}{3}\right\}$

E) $\left\{n\pi\pm\frac{\pi}{6};n\in\mathbb{Z}\right\}$

Recordar!

$\tan\alpha\in\mathbb{R} \longleftrightarrow \alpha\neq(2n+1)\frac{\pi}{2};\ n\in\mathbb{Z}$

Resolución

$\tan^3x-\tan^2x-3\tan x+3=0$; $\tan x>1$

$\tan^2x(\tan x-1)-3(\tan x-1)=0$

$(\tan x-1)(\tan^2x-3)=0$; $\tan x-1>0$

$\tan^2x-3=0 \longrightarrow \tan^2x=3$

$\longrightarrow \tan x=+\sqrt{3}$

$x=n\pi+\arctan(\sqrt{3})$; $n\in\mathbb{Z}$

$\pi/3$

$\therefore CS=\left\{n\pi+\frac{\pi}{3};n\in\mathbb{Z}\right\}$

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PROBLEMA 4

Resuelve la siguiente inecuación

$$(\operatorname{sen}2x-\cos x)\operatorname{sen}\left(\frac{2x}{1+x^2}\right)\geq 0$$

donde $0\leq x\leq \pi$.

A) $\left[\frac{\pi}{6};\frac{\pi}{2}\right]$

B) $\left\langle\frac{\pi}{2};\frac{5\pi}{6}\right\rangle$

C) $\left[\frac{\pi}{6};\frac{5\pi}{6}\right]$

D) $\left[\frac{\pi}{6};\frac{\pi}{2}\right]\cup\left[\frac{5\pi}{6};\pi\right]\cup\{0\}$

E) $\left[0;\frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6};\pi\right]$

$x+\frac{1}{x}\geq 2\ ;\ x>0 \rightarrow 0<\frac{1}{x+\frac{1}{x}}\leq\frac{1}{2}$

$\rightarrow 0<\frac{2x}{x^2+1}\leq 1$, $\in IC$

Resolución

$(\operatorname{sen}2x-\cos x)\operatorname{sen}\left(\frac{2x}{1+x^2}\right)\geq 0\ ;\quad 0\leq x\leq\pi$

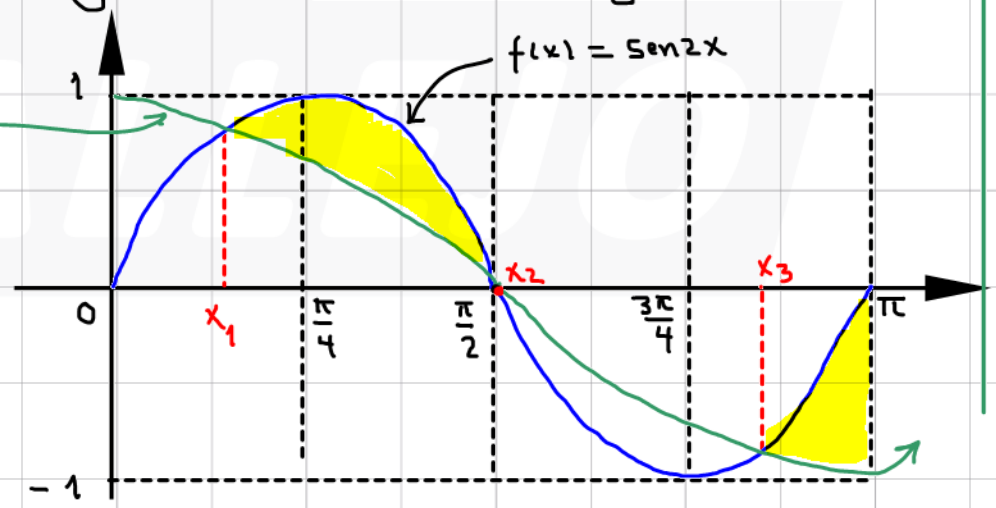
$\operatorname{sen}\left(\frac{2x}{1+x^2}\right)\geq 0$ $\rightarrow$ $\operatorname{sen}\left(\frac{2x}{x^2+1}\right)=0$ $\rightarrow x=0$

$\operatorname{sen}2x-\cos x\geq 0$

$\operatorname{sen}2x \geq \cos x$ $\rightarrow$ $x\in\left[x_1;\frac{\pi}{2}\right]\cup\left[x_3;\pi\right]$

$f(x)$ $\quad$ $g(x)$

- $f(x)=\operatorname{sen}2x \rightarrow T_f=\frac{2\pi}{2}=\pi$
- $g(x)=\cos x \rightarrow T_g=2\pi$



$f(x)=g(x)$

$\operatorname{sen}2x=\cos x$

$2\operatorname{sen}x\cos x=\cos x$

$\cos x=0 \vee \operatorname{sen}x=\frac{1}{2}$

$x=\frac{\pi}{2}\ ;\ x=\frac{\pi}{6};\frac{5\pi}{6}$ (x_1, x_3)

$\therefore CS=\left[\frac{\pi}{6};\frac{\pi}{2}\right]\cup\left[\frac{5\pi}{6};\pi\right]\cup\{0\}$

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PROBLEMA 5

Determine la suma de soluciones de la siguiente ecuación en $0<x<\pi$.

$$\text{sen}x(1+\cos 2x)+\cos x\,\text{sen}2x=1+\cos 2x$$

A) $\frac{3\pi}{2}$ B) π C) 2π

D) $\frac{\pi}{2}$ E) 0

Resolución

$$\text{Sen}x+\underbrace{\text{sen}x\cos 2x+\cos x\,\text{sen}2x}_{\text{Sen}(x+2x)}=1+\cos 2x;\ 0<x<\pi$$

$$\text{Sen}x+\text{Sen}3x=1+\cos 2x$$

$$2\underbrace{\text{Sen}2x}_{2\text{Sen}x\cos x}\cos x=1+\cos 2x$$

$$4\text{Sen}x\cos^2 x=2\cos^2 x \longrightarrow \cos^2 x=0 \ \vee\ 4\text{Sen}x=2$$

$$\cos x=0 \quad \vee \quad \text{Sen}x=\frac{1}{2}$$

$$x=\frac{\pi}{2} \qquad x=\frac{\pi}{6};\frac{5\pi}{6}$$

$$\therefore \text{ Suma de soluciones}=\frac{\pi}{2}+\frac{\pi}{6}+\frac{5\pi}{6}=\frac{3\pi}{2}$$

PROBLEMA 6

Resuelva la siguiente ecuación

$$\operatorname{arcsen} x - \arccos x = \operatorname{arcsen}\left(\frac{1}{2}\right)$$

A) $\left\{\frac{1}{2}\right\}$ B) $\left\{\frac{\sqrt{3}}{2}\right\}$ C) $\left\{\frac{\sqrt{3}}{2}; -\frac{\sqrt{3}}{2}\right\}$

D) $\left\{-\frac{\sqrt{3}}{2}\right\}$ E) $\left\{\frac{1}{2}; -\frac{1}{2}\right\}$

Resolución

$$\underbrace{\operatorname{arcsen} x}_{\frac{\pi}{2} - \operatorname{arcsen} x} - \arccos x = \underbrace{\operatorname{arcsen}\frac{1}{2}}_{\frac{\pi}{6}}, \quad -1 \leq x \leq 1$$

$$\operatorname{arcsen} x = \frac{\pi}{3} \longrightarrow x = \operatorname{sen}\frac{\pi}{3}$$

$$\longrightarrow x = \frac{\sqrt{3}}{2}$$

$$\therefore CS = \left\{\frac{\sqrt{3}}{2}\right\}$$

PROBLEMA 7

Determine el número de soluciones de la siguiente ecuación en $0<x<2\pi$.

$$\sqrt{1+\operatorname{sen}x}+\sqrt{1-\operatorname{sen}x}=\sqrt{2+\operatorname{sen}x}$$

A) 2 B) 1 C) 4
D) 6 E) 3

$-1 \leq \operatorname{Sen}\theta \leq 1 \rightarrow 0 \leq 1+\operatorname{sen}\theta \leq 2$

$1 \geq -\operatorname{Sen}\theta \geq -1 \rightarrow 2 \geq 1-\operatorname{Sen}\theta \geq 0$

$1 \pm \operatorname{sen}\alpha = \left(\operatorname{Sen}\frac{\alpha}{2} \pm \cos\frac{\alpha}{2}\right)^2$

Resolución

Elevamos al cuadrado

$$1+\operatorname{Sen}x+1-\operatorname{sen}x+2\sqrt{1+\operatorname{sen}x}\sqrt{1-\operatorname{sen}x}=2+\operatorname{Sen}x$$

$$2\sqrt{\cos^2 x}=\operatorname{sen}x \longrightarrow 2|\cos x|=\operatorname{sen}x \;;\; \operatorname{sen}x>0$$

$0<x<\pi$

$$\longrightarrow \operatorname{Tan}x=\pm 2$$

$x \in IC \vee IIC$

$\therefore$ 2 soluciones!

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GRACIAS

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